EC 4210 Solutions

Assignment 5

16.4. Consider a silicon photoconductor with the dimensions as shown in Fig. 6.5 irradiated by light. The index of refraction of the material is 1.5.

a. Calculate the fraction of the incident power that will be absorbed by the device at 800 nm. ... at 1 μm?

b. Using Fig. 17.6 on page 262 of these notes, calculate the device transit time for a voltage value of 400 volts at T= 298K.

c. Suppose that the carrier lifetime is 20 ns. Calculate the output current that would be expected for a 400 volt bias when irradiated by 1 pW of laser light at 632.8 nm. The quantum efficiency of the device at the wavelength of interest is assumed to be 60%.

d. Find the normalized mean-square noise current (in units of $\mathrm{A}^2/\mathrm{Hz}^{-1}$) associated with the generation-recombination noise for a direct detection test signal at 100 MHz.

e. Calculate the mean-square signal current for $P_s = 1$ pW with $m = 1$.

f. Calculate the resulting signal-to-noise ratio *in dB* if the noise bandwidth is 10 MHz.

g. Calculate the minimum detectable power for direct detection with this detector if $B = 10$ MHz.

h. Calculate the minimum detectable power for heterodyne detection with this detector.

Solution: a. We need the reflection coefficient.

$$R = \left(\frac{n-1}{n+1}\right)^2 = \left(\frac{1.5-1}{1.5+1}\right)^2 = \left(\frac{0.5}{2.5}\right)^2 = 0.04\,. \tag{1}$$

We know from our previous work that

$$\frac{P(w)}{P_{\text{inc}}} = (1-R)e^{-\alpha d}(1-e^{-\alpha w})\,. \tag{2a}$$

We have $d = 0$, $R = 0.04$ and $w = 5 \times 10^{-3}$. For $\lambda = 800$ nm, we estimate our absorption coefficient from Fig. 16.2 on page 247 as $\alpha = 2 \times 10^5$ m^{-1}, so

$$\frac{P(w)}{P_{\text{inc}}} = 0.96(1)(1 - e^{-(2\times10^5)(5\times10^{-3})}) = 0.96\,. \tag{2b}$$

So, 96% of the power that is not reflected is absorbed.

For $\lambda = 1 \times 10^{-6}$, we estimate our absorption coefficient from Figure 16.2 as $\alpha = 1.8 \times 10^4$ m^{-1}, **Revised 3/2/99** so

$$\frac{P(w)}{P_{\text{inc}}} = 0.96(1)(1 - e^{-(1.8\times 10^4)(5\times 10^{-3})}) = 0.96\,. \tag{2c}$$

The slightly decreased absorption coefficient did not change the fraction of light absorbed.

b. We know that $E = V/l$. At 400 volts, we have

$$E = \frac{V}{l} = \frac{400}{1} = 400 \text{ V}\cdot\text{cm}^{-1}\,. \tag{3}$$

From the carrier velocity curve (Fig. 17.6 on page 262), we find that that the velocity of the holes is about 2×10^5 cm/s ($= 2\times 10^3$ m/s) and that of electrons is about 6.5×10^5 cm/s ($= 6.5\times 10^3$ m/s). From this data, we calculate the average speed of a carrier as

$$< v > = \frac{2\times 10^3 + 6.5\times 10^3}{2} = 4.25\times 10^3 \text{ m}\cdot\text{s}^{-1}\,. \tag{4}$$

The transit time τ_d is

$$\tau_d = \frac{l}{< v >} = \frac{1\times 10^{-2}}{4.25\times 10^3} = 2.35\times 10^{-6} \text{ s} = 2.35\ \mu\text{s}\,. \tag{5}$$

c. Let $V = 400$ and we know that $\tau_0 = 20\times 10^{-9}$ s. We irradiate with 1×10^{-6} at $\lambda = 632.8\times 10^{-9}$ with a quantum efficiency of $\eta = 0.60$. The absorbed power is

$$P_{\text{abs}} = 0.96 P_{\text{inc}} = 0.96\times 10^{-12} \text{ W} \tag{6}$$

from part a. The dc current, then, is

$$\begin{aligned}\bar{i} &= \frac{Pq\eta\lambda}{hc}\frac{\tau_0}{\tau_d} \qquad (7)\\ &= \left(\frac{(0.96\times 10^{-12})(1.6\times 10^{-19})(0.60)(632.8\times 10^{-9})}{(6.63\times 10^{-34})(3.0\times 10^8)}\right)\left(\frac{(20\times 10^{-9})}{(2.35\times 10^{-6})}\right) = 2.49\times 10^{-15} \text{ A}\,.\end{aligned}$$

d. The signal frequqncy is $\omega_s = 2\pi(1\times 10^8) = 6.28\times 10^8$ Hz. The normalized mean-square noise current associated with the generation–recombination noise is

$$\begin{aligned}\frac{< i_N^2 >}{B} &= \frac{4\bar{i}q\left(\frac{\tau_0}{\tau_d}\right)}{1+\omega_s^2\tau_0^2} = \frac{4(2.49\times 10^{-15})(1.6\times 10^{-19})\left(\frac{20\times 10^{-9}}{2.35\times 10^{-6}}\right)}{1+(6.28\times 10^8)^2(20\times 10^{-9})^2} \qquad (8)\\ &= 8.52\times 10^{-38}\text{A}^2\cdot\text{Hz}^{-1}\,.\end{aligned}$$

e. We want to find the mean-square signal current when $P = 1\times 10^{-12}$ W. We first need to find the power absorbed by including the reflection loss.

$$P_{\text{abs}} = 0.96 P_{\text{inc}} = 0.96\times 10^{-12} \text{ W}\,. \tag{9}$$

$$
\begin{aligned}
< i_s^2 > &= \left(\frac{Pq\eta\lambda}{hc}\right)^2 \left(\frac{\tau_0}{\tau_s}\right)^2 \left(\frac{2m^2}{1+\omega_s^2\tau_0^2}\right) \qquad (10)\\
&= \left(\frac{(0.96\times 10^{-12})(1.6\times 10^{-19})(0.60)(632.8\times 10^{-9})}{(6.63\times 10^{-34})(3.0\times 10^{8})}\right)^2 \left(\frac{20\times 10^{-9}}{2.35\times 10^{-6}}\right)^2\\
&\quad \times \left(\frac{2(1)^2}{1+(2\pi\times 1\times 10^{8}\times 20\times 10^{-9})^2}\right)\\
&= 7.88\times 10^{-32}\ \mathrm{A}^2\,.
\end{aligned}
$$

f. For $B = 1\times 10^7$ Hz, we have

$$
\frac{\mathrm{S}}{\mathrm{N}} = \frac{< i_s^2 >}{\frac{< i_n^2 >}{B}B} = \frac{7.88\times 10^{-32}}{(8.54\times 10^{-38})(1\times 10^{7})} = 0.0923 \Rightarrow -10.35\ \mathrm{dB}\,. \qquad (11)
$$

Not very detectable!!

g. The minimum detectable power for direct detection with a photoconductor is

$$
\begin{aligned}
P_{\mathrm{s\ min}} &= \frac{3hcB}{\eta\lambda} = \frac{(3)(6.63\times 10^{-34})(3.0\times 10^{8})(1\times 10^{7})}{(0.60)(632.8\times 10^{-9})} \qquad (12)\\
&= 1.570\times 10^{-11}\ \mathrm{W} = 15.70\ \mathrm{pW}\,.
\end{aligned}
$$

h. The minimum detectable power for heterodyne detection with a photoconductor is

$$
\begin{aligned}
P_{\mathrm{min}} &= \frac{2hcB}{\eta\lambda_L} = \frac{(2)(6.63\times 10^{-34})(3.0\times 10^{8})(1\times 10^{7})}{(0.60)(632.8\times 10^{-9})} \qquad (13)\\
&= 1.049\times 10^{-11}\ \mathrm{W} = 10.49\ \mathrm{pW}\,.
\end{aligned}
$$

16.5 Find the minimum detectable power that a detector with a quantum efficiency of 1 will have when used in heterodyne detection with a 500 nm source and a 1 Hz nominal bandwidth.

Solution: The minimum detectable power for heterodyne detection with a photoconductor is given by Eq. 16.23 as

$$
P_{\mathrm{s\ min}} = \frac{2hcB}{\eta\lambda_L} = \frac{(2)(6.63\times 10^{-34})(3.0\times 10^{8})}{(1)(500\times 10^{-9})} = 7.97\times 10^{-19}\ \mathrm{W}\,. \qquad (14)
$$

17.1. Consider a silicon photodiode with uniform acceptor doping concentration of 1×10^{21} atoms/m^3 and a uniform donor doping of 5×10^{21} atoms/m^3.

a. Calculate V_d if $n_i = 1.8\times 10^{16}$ atoms/m^3 at room temperature. (Note: $kT/q = 0.0259$ volts at room temperature (300K).)

b. Calculate l_p, l_n, and w for 0 volts of reverse bias.

c. For 10 volts of reverse bias?

d. For 100 volts of reverse bias?

e. Find the value of the maximum electric field inside the crystal for 10 volts of reverse bias.

f. Find the device capacitance at 10 volts reverse bias if the circular detector's diameter is 1 mm.

Solution: a. Given that $n_i = 1.8 \times 10^{16}$ and $kT/Q = 0.0259$ volts at room temperature, we find V_d as

$$V_d = \frac{kT}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right) = 0.0259 \ln\left(\frac{(1 \times 10^{21})(5 \times 10^{21})}{(1.8 \times 10^{16})^2}\right) = 0.608 \text{ volts}\,. \tag{15}$$

b. For $V_a = 0$ volts we find

$$\begin{aligned} l_n &= \left(\sqrt{V_a + V_d}\right)\left(\sqrt{\frac{2\epsilon}{q}}\right)\left(\sqrt{\frac{N_A}{N_D(N_A + N_D)}}\right) \qquad \text{(16a)} \\ &= \left(\sqrt{0 + 0.608}\right)\left(\sqrt{\frac{2(1.044 \times 10^{-12})}{1.6 \times 10^{-19}}}\right)\left(\sqrt{\frac{1 \times 10^{21}}{(5 \times 10^{21})(6 \times 10^{21})}}\right) \\ &= 1.626 \times 10^{-8} \text{ m} = 16.26 \text{ nm}\,, \end{aligned}$$

and

$$\begin{aligned} l_p &= \left(\sqrt{V_a + V_d}\right)\left(\sqrt{\frac{2\epsilon}{q}}\right)\left(\sqrt{\frac{N_D}{N_A(N_A + N_D)}}\right) \qquad \text{(16b)} \\ &= \left(\sqrt{0 + 0.608}\right)\left(\sqrt{\frac{2(1.044 \times 10^{-12})}{1.6 \times 10^{-19}}}\right)\left(\sqrt{\frac{5 \times 10^{21}}{(1 \times 10^{21})(6 \times 10^{21})}}\right) \\ &= 8.13 \times 10^{-8} \text{ m} = 81.3 \text{ nm}\,. \end{aligned}$$

Adding the two depletion lengths, we find

$$w = l_n + l_p = 16.26 + 81.3 = 97.5 \text{ nm}\,. \tag{16c}$$

c. For $V_a = 10$ volts, we find

$$l_n(10 \text{ volts}) = l_n(0 \text{ volts})\sqrt{\frac{V_a + V_d}{V_d}} = 16.26\sqrt{\frac{10.61}{0.608}} = 67.9 \text{ nm} \tag{17a}$$

and

$$l_p(10 \text{ volts}) = l_p(0 \text{ volts})\sqrt{\frac{V_a + V_d}{V_d}} = 81.3\sqrt{\frac{10.61}{0.608}} = 339 \text{ nm}\,. \tag{17b}$$

Adding the two depletion lengths, we find

$$w = l_n + l_p = 67.9 + 339 = 408 \text{ nm}\,. \tag{17c}$$

d. For $V_a = 100$ volts we find

$$l_n(100 \text{ volts}) = l_n(0 \text{ volts})\sqrt{\frac{V_a + V_d}{V_d}} = 16.26\sqrt{\frac{100.6}{0.608}} = 209 \text{ nm}\,, \tag{18a}$$

and

$$l_p(100 \text{ volts}) = l_p(0 \text{ volts})\sqrt{\frac{V_a + V_d}{V_a}} = 81.3\sqrt{\frac{100.6}{0.608}} = 1045 \text{ nm}\,. \tag{18b}$$

Adding the two depletion lengths, we find

$$w = l_n + l_p = 209 + 1045 \text{ nm} = 1.255\ \mu\text{m}\,. \tag{18c}$$

e. The value of $E_{\max}$ for $V_a = 10$ volts is

$$E_{\max} = \frac{2(V_d + V_a)}{w} = \frac{2(10.61)}{406 \times 10^{-9}} = 52.1 \times 10^6 \text{ V} \cdot \text{m}^{-1}\,. \tag{19}$$

f. The device capacitance is

$$C_d = \frac{\epsilon A}{w} = \frac{(1.044 \times 10^{-12})(\frac{\pi}{4})(1 \times 10^{-3})^2}{408 \times 10^{-9}} = 2.01 \times 10^{-9} = 2.01 \text{ pF}\,. \tag{20}$$

17.2. Consider a silicon photodiode with a depletion layer that begins 5 μm below the top surface of the detector. Using the results of Problem 2 on Photoconductors, calculate the fraction of the incident light that will be absorbed in the depletion layer (at $\lambda = 0.6\mu$m) for ...

a. ... a pn junction that is 1 μm thick (i.e., $w = 1\mu$m).

b. ... a pin junction that is 5 μm thick.

Solution: We know that

$$\frac{P(w)}{P_{\text{inc}}} = (1 - R)\left(e^{-\alpha d}\right)\left(1 - e^{-\alpha w}\right) \tag{21a}$$

and that

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = \left(\frac{1.5 - 1}{1.5 + 1}\right)^2 = 0.04\,, \tag{21b}$$

so

$$1 - R = 0.96\,. \tag{21c}$$

a. From Fig. 16.2 on p. 249, we estimate the absorption coefficient for slicon at $\lambda = 0.6 \times 10^{-6}$ as $\alpha = 8.0 \times 10^5$ m^{-1}. For $w = 1 \times 10^{-6}$ m, we find **Revised. 2/25/99**

$$\frac{P(w)}{P_{\text{inc}}} = (0.96)(e^{-(8\times10^5)(5\times10^{-6})})\left(1 - e^{-(8\times10^5)(1\times10^{-6})}\right) = 0.00968 \Rightarrow 0.968\% \tag{22a}$$

b. For $w = 5 \times 10^{-6}$, we find

$$\frac{P(w)}{P_{\text{inc}}} = (0.96)(e^{-(8\times10^5)(5\times10^{-6})})\left(1 - e^{-(8\times10^5)(5\times10^{-6})}\right) = 0.01758 \Rightarrow 1.758\% \,. \tag{22b}$$

The increased depletion layer results in almost twice as much light being absorbed.

17.3. It is desired to operate a silicon photodiode in direct detect at a maximum frequency of 100 MHz with a 50Ω load resistance.

a. Calculate A/w for the diode to meet this specification.

b. Calculate the maximum transit time allowed for this device. Assuming that the carriers reach scattering-limited velocities of 6×10^6 cm/s, calculate the maximum width of the depletion layer.

c. Using the results of parts a and b, calculate the maximum diameter that a circular diode can have and still meet the frequency specifications.

Solution: a. The maximum frequency is given by $f_{\text{max}} = 1/2\pi R_L C_d$ and the allowed device capacitance is $C_d = 1/2\pi R_L f_{\text{max}} = \epsilon A/w$, so

$$\frac{A}{w} = \frac{1}{2\pi R_L f_{\text{max}}\epsilon} = \frac{1}{2\pi(50)(1\times10^8)(1.044\times10^{-12})} = 30.5 \text{ m}\,. \tag{23}$$

b. The carriers move with an average scattering-limited velocity of $< v >= 6 \times 10^4$ m·s^{-1}, then $\omega_{\text{max}}\tau_d \ll 1 = 0.1$ and $\tau_d = w/ < v >= 0.1/\omega_{\text{max}}$. The width w is found from

$$w = \frac{0.1 \ < v >}{\omega_{\text{max}}} = \frac{(0.1)(6\times10^4)}{2\pi(1\times10^8)} = 9.55\times10^{-6} = 9.55 \ \mu\text{m}\,. \tag{24}$$

c. We want to find the the maximum diameter D of a circular photodiode, using the results of parts a and b. We know that $A/w = 30.5$ with $w = 9.55 \times 10^{-6}$, so

$$A = 30.5w = (30.5)(9.55\times10^{-6}) = 2.91\times10^{-4}\,. \tag{25a}$$

Since $A = \pi D^2/4$, we have

$$D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(2.91\times10^{-4})}{\pi}} = 1.926\times10^{-2} = 1.926 \text{ cm}\,. \tag{25b}$$